ANALYSIS

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FACTS AND PROPOSITIONS

By Moritz Schlick

WHEN in the spring of last year I wrote my short paper on "Das Fundament der Erkenntnis," sitting leisurely on a balcony overlooking the blue bay of Salerno, it did not occur to me that I was arousing a violent discussion of the "logical positivists' theory of truth." I regarded my little article as nothing but a gentle warning of a true empiricist against certain tendencies towards what seemed to me a rather dogmatic or rationalistic formulation of positivistic principles. I was, therefore, a little surprised when, on account of that paper, I was accused of being a metaphysician and a poet. Finding it impossible, however, to take this indictment seriously, I was neither shocked by the one nor flattered by the other and did not intend to take up the discussion. I hoped my article would speak for itself in spite of the objections raised.

The appearance of Dr. Hempel's clever article in "Analysis" (vol 2, no 4) has changed my mind. He has shaped his arguments clearly and nicely, and I feel it my duty to point out once more, as simply as possible, why I cannot be satisfied with some of the views professed by some of my friends. I shall restrict myself to what seems to me to be the most critical point, i.e. the relationship of "propositions" to "reality."

I have been accused of maintaining that statements can be compared with facts. I plead guilty. I have maintained this. But I protest against my punishment : I refuse to sit in the seat of the metaphysicians. I have often compared propositions to

facts ; so I had no reason to say that it couldn't be done. I found, for instance, in my Baedeker the statement : " This cathedral has two spires," I was able to compare it with " reality " by looking at the cathedral, and this comparison convinced me that Baedeker's assertion was true. Surely you cannot tell me that such a process is impossible and that there is a detestable metaphysics involved in it. You say you did not mean it that way ? But I assure you that I meant nothing but a process of this kind when I spoke of testing propositions by comparing them with facts. No one who is familiar with my recent writings can suppose that I used the term " reality " for anything but empirical objects like churches, trees, clouds etc. (see e.g. " Les énoncés scientifiques," chap. IV, Paris 1934), not for any " metaphysical " entities. A cathedral is not a proposition or a set of propositions, therefore I felt justified in maintaining that a proposition could be compared with reality.

Perhaps you say : " But if we analyse the process of verification of Baedeker's assertion we shall find that it amounts to a comparison of propositions." I answer : I don't know ; it will depend on what you mean by ' analysis.' But whatever the result of your analysis may be, at any rate we can distinguish between cases in which a written, printed or spoken sentence is compared with some other written, printed or spoken sentence, and cases like our example, where a sentence is compared with the thing of which it speaks. And it is this latter case which I took the liberty of describing as a ' comparison of a proposition with a fact.' If you don't like to use this expression for the case described it will merely be a difference of terminology.

You insist that a statement cannot or must not be compared to anything but statements. But why ? It is my humble opinion that we can compare anything to anything if we choose. Do you believe that propositions and facts are too far removed from each other ? Too different ? Is it a mysterious property of propositions that they cannot be compared with anything else ? That would seem to be a rather mystical view. We are assured that the " cleavage " between statements and facts " is nothing but the result of a redoubling metaphysics " (p. 51). That may be true. But who believes in such a cleavage ? Those who say that the one cannot be compared to the other, or the humble empiricist like myself, for whom propositions are facts among other facts

and who sees no difficulty, no " embarrassing consequences " in comparing them ?

What is a proposition, after all ? In my opinion it is a series of sounds or other symbols (a ' sentence ') *together with the logical rules belonging to them*, i.e. certain prescriptions as to how the sentence is to be *used*. These rules, culminating in ' deictic ' definitions, constitute the ' meaning ' of the proposition. In order to verify the proposition I have to ascertain whether those rules have actually been obeyed—why should that be impossible ? In our example it is done by looking at the cathedral and at the sentence in the book and by stating that the symbol ' two ' is used in connection with the symbol ' spires,' and that I arrive at the same symbol when I apply the rules of counting to the towers of the cathedral.

You will reproach me for using again the ' material mode of speech ' instead of the ' formal mode.' I fully acknowledge the importance of the distinction between these two, and I have admitted that in epistemological analysis we should endeavour to use the formal mode. But I think it is wrong to say that the use of this mode is " much more correct." I am convinced, with Carnap, that the material mode as such is not faulty, only it is apt to engender pseudo-problems *if it is employed without sufficient precaution*. Dr. Hempel says : " Saying that empirical statements ' express facts ' . . . is a typical form of the material mode of speech." Perhaps so ; but what harm is there in it ? This particular phrase is an innocent tautology—for what on earth could statements express except facts ? On the other hand : saying that certain black marks in my Baedeker express the fact that a certain cathedral has two spires is a perfectly legitimate empirical assertion. By the way, it is easy to express in a purely " formal " way my opinion that facts and propositions can be compared : words denoting symbols and words denoting other things may occur in the same sentence.

Sometimes we are told that it is in " a logical respect " that propositions cannot be compared to anything but propositions. This may be true, but I don't know whether it is true or not, because I don't know what is meant by a comparison in a " logical respect."

Is it true that we are unable " to give a precise account of how a comparison between statements and facts may possibly be

accomplished, and how we may possibly ascertain the structure of facts" ? (p. 51) I think it is not true. Or was the description faulty which I gave of such a comparison a little while ago ? It consisted of the simplest empirical prescriptions of a kind which we carry out many times almost every day. You can easily make the description more precise by adding details, but that is not even necessary, because it would not change the principle of the matter.

If you assert, as you seem to do, that we cannot "possibly ascertain the structure of facts," I must confess that such a statement reminds me a little of the metaphysics of "things in themselves" which, it is alleged, must for ever remain unknown to us. Since you do not deny the existence of facts (p. 54), why should you deny that we can know their structure ? I should say, for instance, that by counting the spires of a cathedral I get acquainted with the structure of a certain fact. Perhaps you merely want to say that it is nonsense to speak of 'structures of facts' at all ? I answer : that may be so if you adopt certain rules for the use of your words, but it is not so if the words are employed in the way in which I used them. It must be remembered that no sentence is meaningful or meaningless *per se*, but only in regard to the definitions and rules which have been stipulated for the use of the words occurring in it.

This applies to the whole issue in discussion. If it is true that "facts" and "propositions" cannot be compared, then those words are used in a way which differs from the way I used them. They have a different meaning, and the dispute is merely verbal.

The simplest way of maintaining the impossibility of the comparison in question would be to say *that there are no "facts"* (in the formal mode ; the rule for the word 'fact' is simply that it shall not be employed at all). But I see no reason why this convenient word should be banished and, if I understand rightly, you do not actually want to go so far.

Perhaps you mean to say that although the facts which we call propositions *may* be compared with other facts, it is actually *never done in science* ? I think that this is true for the purely theoretical work of science, e.g. for the mathematical physicist whose business consists in formulating and comparing natural laws and also "protocol statements," uniting them into a coherent system and computing the consequences. His work

is done by means of pencil and paper. But I assert most emphatically that it is *not* true for the experimenting scientist whose work consists in making observations and comparing the predictions of the mathematician with—I do beg your pardon—the observed facts.

It is at this point, I believe, that the psychological source of the criticised view reveals itself : its advocates are theoretically minded men who take their stand *within* science. Science is a system of propositions ; and—without being aware of it—these thinkers substitute science for reality ; for them facts are not acknowledged before they are formulated in propositions and taken down in their notebooks. But Science is not the World. The universe of discourse is not the whole universe. It is a typical rationalistic attitude which shows itself here under the guise of the most subtle distinctions. It is as old as metaphysics itself, as we may learn from a saying of old Parmenides, which runs: *ταὐτὸν δ' ἐστὶ νοεῖν τε καὶ οὕνεκεν ἐστι νόημα. . . .*

Our good friends and opponents think of the system of truths as the mathematician thinks of theoretical physics : for him it is quite true that his only task is to make all scientific statements *coherent* among each other ; and it is also true that if there are several coherent systems his choice of the " true " one is solely determined by " the scientists of his culture circle " (p.57) ; he has no other canon because they furnish the " protocol statements " which he uses as his material without submitting them to an experimental test. It is true, therefore, that the system of protocol statements which he calls " true " is " the system which is actually adopted by mankind."

But the matter is different for the experimenting observer and for unrelenting empiricists like myself. It is one thing to ask how the system of science has been built up and why it is generally believed to be true, and another thing to ask why I myself (the individual observer) accept it as true. You may regard my article on " Das Fundament der Erkenntnis " as an attempt to answer the last question. It is a psychological question. If anyone should tell me that I believe in the truth of science ultimately because it has been adopted " by the scientists of my culture circle," I should—smile at him. I do have trust in those good fellows, but that is only because I always found them to be trustworthy wherever I was able to test their enunciations. I

assure you most emphatically that I should *not* call the system of science true if I found its consequences incompatible with my own observations of nature, and the fact that it is adopted by the whole of mankind and taught in all the universities would make no impression on me. If all the scientists in the world told me that under certain experimental conditions I must see three black spots, and if under those conditions I saw only one spot, no power in the universe could induce me to think that the statement " there is now only one black spot in the field of vision " is false.

In other words : the *only ultimate* reason why I accept any proposition as true is to be found in those simple experiences which may be regarded as the final steps of a comparison between a statement and a fact and which I have spoken of as " Konstatierungen "—without attaching any importance to the word. One can perhaps give a better description of them than I have done, but no one can convince me that they are not the only ultimate basis of all my beliefs.

IS STRICT IMPLICATION THE SAME AS ENTAILMENT ?

By AUSTIN E. DUNCAN-JONES

1 Entailment and the modal functions

In this note I shall try to show how strict implication, as developed by C. I. Lewis (in *symbolic logic*, Lewis and Langford), differs from entailment, and to suggest certain requirements which a calculus of entailment would have to satisfy.[1]

Consider the two expressions (1) $x\mathrm{R}y.y\mathrm{R}z$ and (2) $x\mathrm{R}z$. Suppose we let x y and z be classes and R the relation of inclusion ; or suppose we let x y and z be propositions, and R the relation of material implication, or strict implication, or entailment : then most people would admit that (2) is materially implied by (1), that (2) follows from (1), and perhaps that (2) is

[1] The general point of view which I am adopting is much the same as that of Everett J. Nelson in *intensional relations*, *Mind* 1930. I had already worked out this paper in outline before I discovered the existence of Nelson's paper : otherwise the note would have been based directly on Nelson. I differ from Nelson on one or two points to be mentioned below.

strictly implied by (1), except that strict implication has not so far as I know been extended to relations of classes. On any of the suggested interpretations (1) and (2) quite clearly satisfy the definition of entailment as given by Moore (*philosophical studies* p. 291) : Moore writes, ' we require . . . some term to express the converse of the relation which we assert to hold between a particular proposition q and a particular proposition p, when we assert that q follows from or is deducible from p. Let us use the term " entails " to express the converse of this relation'. As examples of what he means by *following* he gives the relation between the conclusion of a syllogism in Barbara and the premisses, taken as one compound proposition ; or the relation between ' this is coloured ' and ' this is red.'

The propositions (1) and (2) also satisfy a condition which I suspect to be part of the analysis of entailment as Moore uses the word, though I have no authority for thinking that Moore would agree to this : the condition I refer to is one which I think anyone can detect by looking at (1) and (2), or at Moore's examples as quoted ; it may be expressed by saying that (2) arises out of the meaning of (1). This, of course, is not a satisfactory expression, but I am afraid it must serve for the present. I propose in this paper to use the word *entailment* for a relation which only holds between two propositions, p and q, when q arises out of the meaning of p ; and I shall symbolise this relation between p and q by $p\mathbf{E}q$.

Closely connected with entailment are consistency and the modal functions. I shall symbolise p *is consistent with* q by $p\mathbf{K}q$; I shall define possibility in terms of consistency, and the other modal functions in terms of possibility ; p *is possible* I shall write $\mathbf{L}p$.

Suppose we try to define $p\mathbf{K}q$ or $\mathbf{L}p$ in terms of $\mathbf{E}$, the obvious way to do it is to define $p\mathbf{K}q$ as $-.p\mathbf{E}-q$, and $\mathbf{L}p$ as $-.p\mathbf{E}-p$ (compare *symbolic logic* df.17.01 and theorem 18.1). But consider such a pair of propositions as p and $-pq$: these are clearly not consistent ; yet it does not seem as though, whatever q may be, p entails the denial of $-pq$. Thus according to the proposed definition p and $-pq$ would be consistent. Again, consider the proposition $p-p$: this is obviously impossible ; but it does not seem as though $p-p$ entails its own denial ; and if not, $p-p$ will be possible according to the proposed definition.

A much less fishy account can be given of logical possibility, in the familiar sense in which it clearly has some close connexion with freedom from contradiction. First, it seems that a proposition is impossible if it is either (1) a conjunction of contradictory propositions—this I shall call a contradictory pair ; or (2) a conjunction of contrary propositions—this I shall call a contrary pair ; or (3) a conjunction of a contradictory pair and some other proposition ; or (4) a conjunction of a contrary pair and some other proposition. A proposition of any of these four forms will entail a contradiction: we may define a possible proposition as one which does not entail a contradiction, and an impossible proposition as one which does. This definition is clearly satisfied by $p \cdot \sim p$, which entails a contradiction—itself—although it does not entail its own contradictory, as it would need to under the first definition considered.

What is meant here by contrary propositions may be shortly explained as follows. Suppose A is a certain determinable character, whose determinates are α_1, α_2, α_3 and so on : $\alpha_1 x$ is a proposition of which α_1 is an element, and $\alpha_2 x$ is a proposition differing from $\alpha_1 x$ only in that it has α_2 as an element instead of α_1. Two propositions will be contrary to one another if they are either of the forms $\alpha_n x$ and $\sim Ax$, or of the forms $\alpha_m x$ and $\alpha_n x$, where m and n are different. It is obvious, I think, that in either of these pairs of propositions either member of the pair entails the contradictory of the other ; and consequently a contrary pair always entails a contradictory pair. For example, $\alpha_m x . \alpha_n x : \mathbf{E} : \alpha_m x . \sim \alpha_m x$. In this symbolism x may be either a name or a definite description or a generalised variable: that is, Ax will be either of the form Ac, where c is a name ; or of the form $A(\iota y)(\phi y)$; or of the form $(y).Ay$, or $(y).\phi y \supset Ay$. Thus, roughly, x is any function of or operation on an α which results in a proposition with that α as an element, such that any proposition is inconsistent with it which differs from it only by the substitution for this α of some other, or by the substitution of A and the negating of the proposition.

Consistency may be defined as the relation which holds between two propositions, p and q, when p does not entail any proposition the denial of which is entailed by q : thus—

$$p \mathbf{K} q \ . \ =\text{df} \ . \ \sim (\exists r) . p \mathbf{E} r . q \mathbf{E} \sim r$$

Since $p \mathbf{E} p$, this definition is not satisfied by p and $\sim pq$. It may

be remarked that $\vdash : p - p . \mathbf{K} . p - p$, but $p - p . \mathbf{E} . - (p - p)$ does not follow. Obviously, from the meaning of $\mathbf{E}$, the relation of consistency is symmetrical.

A formal definition of possibility, satisfying the condition that a proposition is possible if it does not entail a contradiction, may now be given in terms of consistency. Writing $\mathbf{L}p$ for *p is possible*,

$$\mathbf{L}p \,.\, = \text{df} \,.\, p\mathbf{K}p$$

The symbolism for impossibility, possible falsity, and necessity, will resemble Lewis'.

$-\mathbf{L}p$ for *p is impossible*

$\mathbf{L}-p$ for *p is possibly false*

$-\mathbf{L}-p$ for *p is necessary*

The need for the reference to existence may be seen by considering the defined equivalent of $-\mathbf{L}p$ (or $-\mathbf{L}-p$): $-\mathbf{L}p$ is $(\exists q).p\mathbf{E}q.p\mathbf{E}-q$. Now if $(\exists q)$ had been left out, we should have had, for $\mathbf{L}p$, $\vdash : p\mathbf{E}q.p\mathbf{E}-q$; that is, for any q, p does not both entail q and entail $-q$. This may be quite correct, but it gives us, for $-\mathbf{L}p$, $p\mathbf{E}q.p\mathbf{E}-q$; that is, for any q, p entails both p and $-q$, that is, an impossible proposition entails any proposition and its contradictory. This, of course, is just the sort of paradox which I am hoping to avoid.

I think it is fairly clear that the definition here suggested does give the ordinary meaning of *logical possibility*, *necessity*, and so on; though in a deductive logical system it might be convenient to introduce modality as a primitive idea rather than define it in terms of existence.

2 Strict implication

Lewis does not claim, in the account of strict implication given in *symbolic logic* (chs. 6 & 8), that strict implication is identical with entailment, as defined by Moore. But the explanation of strict implication which he gives closely resembles Moore's account of entailment. He writes (op. cit. ch. 8, p. 247): 'it appears that the relation of strict implication expresses precisely that relation which holds when valid deduction is possible, and fails to hold when valid deduction is not possible.' Throughout this chapter arguments are used the purpose of which seems to be to show that 'p strictly implies q' is equivalent to 'q is deducible from p.' It is not necessary for me to prove that Lewis means by

strict implication what Moore means by entailment : for in any case the identification is very natural, and it is worth while to consider whether it can correctly be made.

It seems clear that many of the theorems in Lewis' system do actually hold if strict implication is read as entailment. But there are certain important theorems which do not hold. I believe it can be shown, though I shall not try to show in detail, that all the rejected theorems are consequences of Lewis' definition of strict implication. I propose to write $p\mathbf{S}q$ for *p strictly implies q*. $p\mathbf{S}q$ is defined by Lewis in terms of possibility. Not much is said about the meaning of *possible*, but I suspect that Lewis is using the word in approximately the sense that I have just defined. For the present, however, I shall write $\mathbf{J}p$ for *p is possible* in Lewis' sense. Lewis' definition of *p strictly implies q* will then be

$p\mathbf{S}q \;.=\text{df}.\; -\mathbf{J}.p-q$, that is, the product of p and $-q$ is impossible (op. cit. th. 11.02).

It can, I think, easily be shown that, if $\mathbf{J}p$ is the same as $\mathbf{L}p$, $p\mathbf{S}q$ holds whenever $p\mathbf{E}q$ holds, and that $p\mathbf{E}q$ holds in most cases in which $p\mathbf{S}q$ holds, but not in all. First, it seems clear that whenever q arises out of the meaning of p, $p-q$ will consist of the joint assertion of the meaning of p and denial of part of the meaning of p : $p-q$ will thus be either a contradictory pair or a product of a contradictory pair and some other proposition, and will thus be impossible. On the other hand, the conditions in which $p\mathbf{S}q$ may hold are not confined to $p\mathbf{E}q$.

Consider such a proposition as $p-pq$. This entails both p and $-p$. Thus we get $p-pq.\mathbf{E}p$: $p-pq.\mathbf{E}-p$. Thus there is a proposition, (namely p) such that $p-pq$ entails both it and its contradictory. Thus $(\exists r):p-pq.\mathbf{E}r:p-pq.\mathbf{E}-r$. Thus, by the definition of $\mathbf{L}$, $-\mathbf{L}.p-pq$. If $\mathbf{J}$ is the same as $\mathbf{L}$, then by the definition of $\mathbf{S}$, and the ordinary associative laws, we obtain six strict implications from $-\mathbf{L}.p-pq$.

(S1)	$pq.\mathbf{S}p$	(S4)	$-p\mathbf{S}.-(pq)$
(S2)	$-pq.\mathbf{S}-p$	(S5)	$p\mathbf{S}.-(-pq)$
(S3)	$p-p.\mathbf{S}-q$	(S6)	$q\mathbf{S}.-(p-p)$

This example carries us straight to the so called paradoxes of strict implication (op. cit. th. 19.72—7), which Lewis describes as 'inescapable consequences of logical principles'—quite rightly : only, like the paradoxes of material implication, they

are not paradoxical unless they are regarded as theorems of entailment.

It will easily be seen, from consideration of these six propositions, that there are at least two distinct ways in which the impossibility of a logical product can arise ; from inconsistency between the propositions conjoined, as in (S1) and (S2), and from impossibility of one of the propositions conjoined, as in (S3) and (S6). The status of (S4) and (S5) I shall consider presently. The first of these alternatives gives us entailment, the second does not : but both alike give us strict implication. Thus $p\mathbf{S}q$ sometimes holds when $p\mathbf{E}q$ does not hold. If one considers entailment in relation to consistency, it appears that while $p\mathbf{E}q$ can only occur when p is inconsistent with $-q$, strict implication can occur both when p is inconsistent with $-q$, and when p and q are consistent but one of them is impossible. Whenever $p\mathbf{S}q$ is true in virtue of the second of these alternatives, the theorem has a paradoxical appearance ; and (S3) and (S6) above are in fact simply special cases of the so called paradoxes, that an impossible proposition strictly implies any proposition, and any proposition strictly implies a necessary proposition. All the paradoxical theorems, in which the assertion of $p\mathbf{S}q$ is based on the necessity of q or the impossibility of p, must be excluded from a calculus of entailment : since it is obvious that $-\mathbf{L}p$ does not entail that every proposition arises out of the meaning of p, and $-\mathbf{L}-p$ does not entail that p arises out of the meaning of every proposition. It does not follow in the least that Lewis' calculus of strict implication is not correctly developed, but only that it is not a calculus of deducibility, though it is very much like one.

3 Paradoxes of entailment ?

Unfortunately a grave danger of paradoxes arising in a system based on entailment has been discovered and pointed out by Nelson (op. cit. pp. 447—8). I shall state the difficulty in my own words. There are two principles which, at first sight, one would say must obviously hold in a calculus of entailment : a principle of what may be called converse entailment.

$p\mathbf{E}q \,.\, \equiv \,.\, -q\mathbf{E}-p$ (where $\equiv$ is the sign of reciprocal entailment), and the principle of the antilogism

$pq\mathbf{E}r \,.\, \equiv \,.\, p-r\mathbf{E}-q \,.\, \equiv \,.\, q-r\mathbf{E}-p$. I seem, in fact, to have assumed the first of these principles in my remarks on

consistency. I have also assumed, in my definition of possibility and my remarks on Lewis, that $pq\mathbf{E}p$. But if we apply to the proposition $pq\mathbf{E}p$ the principles of converse entailment and antilogism we get, by a series of transformations, the following six propositions.

(E1)	$pq.\mathbf{E}p.$	(E4)	$-p\mathbf{E}.-(pq).$
(E2)	$-pq.\mathbf{E}-p.$	(E5)	$p\mathbf{E}.-(-pq).$
(E3)	$p-p.\mathbf{E}-q.$	(E6)	$q\mathbf{E}.-(p-p)$

These propositions correspond precisely to the six theorems of strict implication stated in the last section, and display, of course, all the objectionable features which the others seemed to display. So I have got to find some means of disallowing (E3) and (E6), and to make up my mind about (E4) and (E5).

Nelson's way out of the difficulty is very radical : to deny $pq\mathbf{E}p$. In anticipation of this procedure, he writes (p.444) : 'I do not take pq to mean "p is true and q is true," but simply "p and q," which is a unit or whole. . . . and expresses the *joint* force of p and q'; 'pq does not entail r unless both p and q function together in entailing r.' The distinction here drawn between pq and 'p is true and q is true' seems to me quite inadmissible. For if conjunction is given its ordinary meaning, as Nelson seems to intend, pq simply means the same as 'pq is true,' which means the same as 'p is true and q is true.' It seems out of the question to deny that a conjunction does entail each of its conjuncts. I shall show presently that in addition the denial is not necessary for the avoidance of paradox.

Yet there clearly is a difference between $pq\mathbf{E}p$ and, for instance, $p.p\mathbf{E}q : \mathbf{E}q$: in the first case, we have something following from a premiss part of which can be dropped ; in the second case nothing can be dropped. It is quite easy to define this further character which appears in the second case. Where we have $p\mathbf{E}q$, and p is not a conjunction which would still entail q if one of its conjuncts were dropped, I shall say, arbitrarily, that p invokes q : I write this relation $p\mathbf{N}q$, and define it as follows.

$$p\mathbf{N}q \;.=\mathrm{df}.\; p\mathbf{E}q.-(\exists r,s).p=rs.r\mathbf{E}q$$

To avoid the paradoxes I have pointed out all we need do is construct a calculus of entailment in which the proposition $pq\mathbf{E}q$ is allowed, in which $\mathbf{N}$ is defined, and in which a large number of the theorems are based on invocation rather than entailment.

Nelson tries to lay the foundations of a calculus of entailment in which propositions of the form $pq\mathbf{E}p$ are excluded altogether. I do not think the attempt is successful : at least two features of it deserve comment.

(1) Nelson points out that the principle of the syllogism, $p\mathbf{E}q.q\mathbf{E}r:\mathbf{E}.p\mathbf{E}r$, can become, by substitution of q for r, $p\mathbf{E}q.q\mathbf{E}q:\mathbf{E}.p\mathbf{E}q$, which is of the form $pq\mathbf{E}q$. To prevent this he uses the prefix $p\ddagger q\ddagger r$ (p q and r are distinct), and writes

$$p\ddagger q\ddagger r.\mathbf{E}:.\ p\mathbf{E}q.q\mathbf{E}r:\mathbf{E}.p\mathbf{E}r.$$

In passing, I must remark that this is an invalid form, since the principle of the syllogism certainly does not follow from the distinctness of p q and r: this flaw would be removed if one wrote instead

$p\ddagger q\ddagger r.p\mathbf{E}q.q\mathbf{E}r:\mathbf{E}.p\mathbf{E}r$. But the need for a prefix only arises on the supposition that the form $pq\mathbf{E}q$ must be excluded.

(2) Nelson gives (p.449) the principle of antilogism as a theorem :

$pq\mathbf{E}r\ .\ =\ .\ p\mathbf{-}r\mathbf{E}\mathbf{-}q\ .\ =\ .\ q\mathbf{-}r\mathbf{E}\mathbf{-}p$. Now there is nothing to prevent the occurrence of $pq\mathbf{E}p$ as an unasserted element in a theorem : thus we may substitute p for r in this proposition, and get the theorem

$pq\mathbf{E}p\ .\ =\ .\ p\mathbf{-}p\mathbf{E}\mathbf{-}q$, which is one of the paradoxes we are trying to avoid.

Thus Nelson excludes a possible type of theorem quite unnecessarily, and does not succeed in excluding paradox. Having defined $\mathbf{N}$, we may admit all propositions of the form $pq\mathbf{E}p$, and admit the principle of the syllogism without any restriction, in the form $p\mathbf{E}q.q\mathbf{E}r:\mathbf{E}.p\mathbf{E}r$. The principles of converse entailment (or converse invocation) and antilogism must be given in the forms

$$p\mathbf{N}q\ .\ =\ .\ \mathbf{-}q\mathbf{N}\mathbf{-}p$$
$$pq\mathbf{N}r\ .\ =\ .\ p\mathbf{-}r\mathbf{N}\mathbf{-}q\ .\ =\ .\ q\mathbf{-}r\mathbf{N}\mathbf{-}p$$

Neither of these laws enables any conclusion to be drawn from $pq\mathbf{E}p$, and the paradoxes are thus avoided. The $=$ sign here is the sign of reciprocal entailment : and, in general, in a calculus of entailment the asserted relation must usually be entailment, not invocation. If a proposition of the form $p\mathbf{N}q$ were asserted it would be likely to have as a consequence the invalid $pq\mathbf{N}p$.

This is perhaps the solution of the technical difficulty of reconciling $pq\mathbf{E}q$ with converse entailment and antilogism. But

there remains the serious and important question : do the propositions (E1) and (E4) above in fact entail one another ? and similarly, do (E2) and (E5) entail one another ? I can only give my answer to this dogmatically. It seems to me that (E4) and (E5) are in fact false, and that since (E1) and (E2) are true the entailments certainly do not hold. It seems to me that only (E1) and (E2) out of the whole set are true, that (E1) entails (E2), not in virtue of the principle of antilogism, but because $-p$ is a value of p, and that (E2) does not entail (E1). (E3) and (E6) have nothing to be said for them in a calculus of entailment : it is only the corresponding **S** principles which give them any colour. (E4) and (E5) do not seem to me outrageous in the way in which (E3) and (E6) are : I think it just possible that a relation wider than **E** and different from **S** could be defined, for which theorems corresponding to (E4) and (E5) would hold, but none corresponding to (E3) and (E6).

University of Birmingham, March, 1935.

THE ANALYSIS OF SENSE-DATA

By J. O. WISDOM

CERTAIN philosophers have given the impression that they considered a visual sense-datum (to be referred to briefly as a " sense-datum ") to be a colour-here-now. Shape however ought to be inserted. I will use " colour " for " absolutely specific shade of colour " and " shape " for " absolutely specific shape."

Is the conception coloured-shape-here-now or is it shaped-colour-here-now ? Neither of these seems to me correct, for neither colour nor shape is a particular, in the sense that there is nothing which either could characterise. If, however, sense-data are to be analysed, some particular (in this sense) must be found. It is, therefore, worth enquiring if the here or the now can be particulars.

The difficulty that confronts us is that both " here " and " now " are adverbs—a point which has, I believe, been overlooked by philosophers. Thus to say that a colour and a shape were here now would be to say something about the manner in

which the colour and the shape existed. If " I am seeing a sense-datum " were translated by " A colour and a shape are here now," something would have been omitted, namely the *particularity* of the sense-datum.

I ought perhaps to mention that all the important words used in this article are sensory words, and not physical or material. Thus " here " is not used of a place whose spatial coordinates might be given but of a place in visual space. " Now " is not used of a moment which is so many seconds past twelve o'clock, but of time as *consciously* apprehended.

Suppose we invent the words " Here " and " Now," spelt with capitals to show they are nouns and different from the adverbs " here " and " now." " Here " refers to an absolutely specific place in visual space, and " Now " to an absolutely specific time—as consciously apprehended ; not clock-time. Can we say that " I am seeing a sense-datum " can be translated by " Here is now characterised by a colour and by a shape " or by " Now is here characterised by a colour and by a shape " ?

I do not think so, because of the implication that Here need not be *now* characterised in these ways and that Now need not be *here* characterised in these ways. It seems to me to follow from the usage of " Here " that Here is necessarily now, and similarly that Now is necessarily here. Accordingly the adverbs here and now are once more misleading, and it is best to resort to the word " Herenow."[1]

The analysis now becomes, *Herenow is characterised by a colour and by a shape.* Herenows seem to be particulars as required.

It should be carefully noted that there is no circle in explaining the usage of " Herenow " by reference to visual space, which in its turn is explained as the space sense-data occupy. This is the psychological order of explanation ; but logically the order is the reverse, and " Herenow " is indefinable.

I want next to draw attention to a certain peculiarity. The word " now," as above used in a sensory sense, is quite unlike " now," which refers to a moment of clock-time, in an important respect. The now to which the latter refers has but momentary existence, whereas " now " in the sensory sense can refer to durations of any length. Restricting " duration " to be used as

[1] " Nowhere " would of course be exactly equivalent, but would scarcely do since it would probably be read " No-where " instead of " Now-here."

a sensory word, Herenow can have a duration corresponding to a lapse of several seconds—theoretically as long or as short as we please. It is only when some interruption occurs that one Herenow succeeds another Herenow. If I gaze dreamily at a sense-datum the Herenow seems to be " present all at once " until I introspect and notice that it was bounded into a duration corresponding to a lapse of several seconds.

An interesting result follows. A sense-datum can be numerically the same for a considerable time, and will be changed only by a change of Herenow. It can, moreover, be changed into an exactly similar sense-datum by a suitable change of Herenow. Thus if I gaze at a sense-datum I become conscious of my heart-beats, an effect which interrupts the Herenows. Thus I obtain a series of exactly similar Herenows and sense-data at intervals of, say, a second. But this very consciousness of the heart-beats seems to prevent any further interruption, so that I feel certain by introspection that the Herenows do last about this time.

It also follows that an unperceived sensibile is self-contradictory (not nonsense unless you count tautologies and contradictions as nonsense), for it would have to exist without the Herenow. Disbelief in unperceived sensibilia has led philosophers to suppose that sense-data were only momentary or existed only during the " specious present." The view I put forward, while not going back to unperceived sensibilia, yet has the advantage, by allowing sense-data and Herenows to have considerable duration (limited not by logical but by causal considerations only), of avoiding that highly dubious entity, the specious present.

The thesis might be summarised thus. It is misleading to talk of the occurrence of a sense-datum at a certain place or time. For statements of this kind disguise the form of the propositions they express. They must be replaced by statements which attribute characteristics to a Herenow. [2]

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[2] I am not clear how much of the above I owe to Professor Stebbing. A remark she made about the possibility of having Absolute Heres and persistent sense-data acted as a stimulus.